

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

1. Give a definition, equivalent to that in the text or from class, for each of the following terms. (7 points each)

(a) *the function f has the limit L at a* (b) *the function f is continuous at a*

2. Simplify $\tan(\cos^{-1} x)$. That is, rewrite $\tan(\cos^{-1} x)$ as an algebraic expression. (6 points)

3. Consider the function formula $f(x) = x^2 - 4$.

(a) Choose the domain for f to be \mathbb{R} (all real numbers). Show that this function does not have an inverse function, (4 points)

(b) Choose the domain for f to be $[0, \infty)$. Find the inverse function. (4 points)

(c) Choose the domain for f to be $(-\infty, 0]$. Find the inverse function. (4 points)

4. Analyze $\lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - 2}{x - 8}$ using a table of input/output pairs. Give enough evidence to conjecture the limit to 3 decimal places. (8 points)

5. For each of the following, evaluate the limit using techniques that give an exact result if possible. Show enough details to make your methods clear to a reader. (6 points each)

(a) $\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$

(d) $\lim_{y \rightarrow 0} \frac{(x + y)^2 - x^2}{y}$

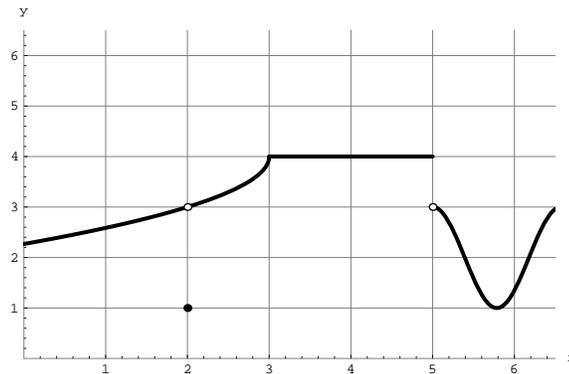
(b) $\lim_{x \rightarrow 2} \frac{x - 2}{x + 3}$

(e) $\lim_{x \rightarrow 2} \frac{x^3 + 1}{x - 1}$

(c) $\lim_{t \rightarrow 0} \frac{t}{\sin(5t)}$

(f) $\lim_{x \rightarrow -3} f(x)$ where $f(x) = \begin{cases} x^2 & \text{if } x < -3, \\ 9 & \text{if } x = -3 \\ 4x + 10 & \text{if } x > -3. \end{cases}$

6. The plot below shows the graph of a function f . Use the graph to analyze continuity of the function for the interval $[0, 6]$. Determine the points at which the function is discontinuous. For each point of discontinuity, state specifically how the definition of continuity fails to hold by analyzing the relevant limits and outputs. (10 points)



7. Consider the function

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ -1 & \text{if } x \text{ is irrational.} \end{cases}$$

Give reasoning to support the claim that $\lim_{x \rightarrow 5} f(x)$ does not exist.

(8 points)

8. The table to the right gives input/output pairs for a function f . Is it possible that the limit of this function as x approaches 2 is *not* equal to 5. Explain how you reach your conclusion. (6 points)

x	$f(x)$
1.9	5
1.99	5
1.999	5
2.001	5
2.01	5
2.1	5